32 ALGEBRAIC FRACTIONS AND ALGEBRAIC PROOF

The word algebra is derived from the Arabic word Al-Jabr. It first appeared in 820AD in the work of the Persian mathematician Al-Khwarizmi. Al-Khwarizmi became known as the father of algebra, creating and using algebraic proofs to solve the mathematical problems of the time.

Objectives

In this chapter you will:

- simplify algebraic fractions
- add and subtract algebraic fractions
- multiply and divide algebraic fractions
- prove a given result using algebra.

Before you start

You need to be able to:

- add, subtract, multiply and divide fractions
- factorise algebraic expressions
- use the laws of indices.

32.1 Simplifying algebraic fractions

Objective

- You can simplify algebraic fractions.

Why do this?

Doctors, engineers and scientists often have to use and simplify algebraic fractions in their jobs.

Get Ready

1. Factorise fully.
   a $x^2 + 5x + 4$ **b** $x^2 - x - 6$ **c** $2x^2 + 7x + 3$

Key Points

- Algebraic fractions, like numerical fractions, can often be simplified.
- To simplify an algebraic fraction, factorise the numerator and the denominator. Then divide the numerator and denominator by any common factors.

Example 1

Simplify fully $\frac{2x^2 + 4x}{x^2 + 3x + 2}$

$2x^2 + 4x = 2x(x + 2)$ ← Factorise the numerator fully.

$x^2 + 3x + 2 = (x + 2)(x + 1)$ ← Factorise the denominator.

$\frac{2x^2 + 4x}{x^2 + 3x + 2} = \frac{2x\cancel{(x + 2)}^1}{{}^1\cancel{(x + 2)}(x + 1)}$ ← Write the fraction in fully factorised form.

$= \frac{2x}{(x + 1)}$ ← Divide both the numerator and denominator by the common factor $(x + 2)$.

$= \frac{2x}{x + 1}$ ← Write your answer without the bracket as there is only one factor in the denominator.

ResultsPlus Watch Out!

You cannot simplify an algebraic fraction until it has been expressed as a product of factors.

Example 2

Simplify fully $\frac{2x^2 - 5x - 3}{x^3 - 9x}$

$2x^2 - 5x - 3 = (2x + 1)(x - 3)$ ← Factorise the numerator.

$x^3 - 9x = x(x^2 - 9)$ ← Factorise the denominator.

$= x(x + 3)(x - 3)$ ← Take out the common factor x. Factorise $(x^2 - 9)$ using the difference of two squares.

$\frac{2x^2 - 5x - 3}{x^3 - 9x} = \frac{(2x + 1)(x - 3)}{x(x + 3)(x - 3)}$ ← Write the fraction in factorised form.

$= \frac{(2x + 1)\cancel{(x - 3)}^1}{x(x + 3)\cancel{(x - 3)}^1}$ ← Divide both the numerator and denominator by any common factors.

$= \frac{2x + 1}{x(x + 3)}$ ← Write the numerator without the bracket as there is only one factor left in the numerator.

Exercise 32A

Questions in this chapter are targeted at the grades indicated.

A

1 Simplify fully.

a $\frac{2x^5}{x^2}$ b $\frac{x^2y}{3xy^2}$ c $\frac{x^2 - 5x}{2x}$ d $\frac{x^2 + 3x}{x + 3}$ e $\frac{2x - 4x^2}{2x - 1}$

A*

2 Simplify fully.

a $\frac{x^2 + 4x + 3}{x^2 + 5x + 6}$ b $\frac{x^2 + 6x + 5}{x^2 + 5x}$ c $\frac{x^2 - 5x + 6}{x^2 + x - 12}$ d $\frac{x^2 - x - 12}{x^2 + 6x + 9}$

3 Simplify fully.

a $\frac{x^2 - 1}{x^2 - x}$ b $\frac{4x^2 + 24x}{x^2 - 36}$ c $\frac{2x^2 - 8}{x^2 + 4x + 4}$ d $\frac{3x^2 - 27}{3x^2 + 9x}$

4 Simplify fully.

a $\frac{2x^2 + 5x + 3}{3x^2 + 5x + 2}$ b $\frac{10x^2 - x - 3}{6x^2 - x - 2}$ c $\frac{9x^2 - 1}{9x^2 - 6x + 1}$ d $\frac{6x^2 + 5x - 1}{12x^2 + 16x - 3}$

5 Simplify fully.

a $\frac{x(x + 5)}{x^2 - 5x}$ b $\frac{x^2 - 10x + 25}{2x^2 - 50}$ c $\frac{8x^2 - 10x + 3}{8x^2 - 6x}$

d $\frac{6x^2 - 2x^3}{2x^3 + 6x^2}$ e $\frac{4 - x^2}{(x + 2)^2}$ f $\frac{16 - x^2}{x - 4}$

32.2 Adding and subtracting algebraic fractions

Objective

- You can add and subtract algebraic fractions.

Why do this?

If you win two competitions in a tennis tournament, and you know what fraction of the prize money each one is worth, you can work out what the total prize money for the tournament was.

Get Ready

1. Write down the lowest common multiple (LCM) of:

a 6 and 15 **b** $3x$ and $4x$ **c** $(x + 1)$ and $x(x + 1)$.

Key Points

- To add (or subtract) algebraic fractions we use a similar method to that used for adding and subtracting numerical fractions.
- If the denominators of the fractions are the same, add (or subtract) the numerators but do not change the denominator.
- To add (or subtract) algebraic fractions with different denominators, find a common denominator and write each fraction as an equivalent fraction with this denominator.
- To find the lowest common denominator of algebraic fractions, you may need to factorise the denominators first.
- To simplify your answers, you may have to factorise the numerator.

Example 3 Add $\frac{2}{x} + \frac{3}{x}$

$\frac{2}{x} + \frac{3}{x} = \frac{5}{x}$ ← The denominators are the same so just add the numerators.

Example 4 Subtract $\frac{5x}{7} - \frac{3x}{7}$

$\frac{5x}{7} - \frac{3x}{7} = \frac{2x}{7}$ ← Subtract the numerators.

Example 5 Write $\frac{3}{2x} - \frac{1}{x}$ as a single fraction.

$\frac{3}{2x} - \frac{1}{x} = \frac{3}{2x} - \frac{2}{2x}$ ← Write each fraction with the same common denominator.

$= \frac{1}{2x}$ ← Subtract the numerators, but leave the denominator the same.

Example 6 Simplify $\frac{x+2}{3} + \frac{x-1}{4}$

Common denominator = 12. ← Work out the lowest common denominator.

$\frac{x+2}{3} + \frac{x-1}{4} = \frac{4(x+2)}{12} + \frac{3(x-1)}{12}$ ← Write as equivalent fractions with the same denominator.

$= \frac{4(x+2) + 3(x-1)}{12}$ ← Add the two fractions.

$= \frac{4x + 8 + 3x - 3}{12}$ ← Expand the brackets.

$= \frac{7x + 5}{12}$ ← Simplify the numerator.

Example 7 Write $\frac{3}{x-1} - \frac{2}{x+1}$ as a single fraction.

Common denominator = $(x-1)(x+1)$ ← Find a common denominator.

$\frac{3}{x-1} - \frac{2}{x+1} = \frac{3(x+1)}{(x-1)(x+1)} - \frac{2(x-1)}{(x-1)(x+1)}$ ← Convert each fraction to an equivalent fraction with the common denominator $(x-1)(x+1)$.

$= \frac{3(x+1) - 2(x-1)}{(x-1)(x+1)}$ ← Subtract the fractions.

$= \frac{3x + 3 - 2x + 2}{(x-1)(x+1)}$ ← Expand the brackets in the numerator.

$= \frac{x + 5}{(x-1)(x+1)}$ ← Simplify the numerator.

ResultsPlus
Watch Out!
There is no need to multiply out the brackets in the denominator.

A

A*

Exercise 32B

1 Write as a single fraction in its simplest form.

a $\frac{2x}{3} + \frac{x}{3}$ b $\frac{x}{2} + \frac{3x}{2}$ c $\frac{3}{10x} + \frac{4}{10x}$

d $\frac{7x}{9} - \frac{3x}{9}$ e $\frac{4x}{5} - \frac{3x}{5}$ f $\frac{7}{3x} - \frac{2}{3x}$

2 Write as a single fraction in its simplest form.

a $\frac{x}{3} + \frac{x}{4}$ b $\frac{x}{5} + \frac{2x}{15}$ c $\frac{x}{2} - \frac{x}{8}$

d $\frac{3x}{2} - \frac{2x}{3}$ e $\frac{1}{3x} + \frac{1}{2x}$ f $\frac{4}{10x} - \frac{3}{20x}$

3 Simplify.

a $\frac{x}{2} + \frac{x+1}{3}$ b $\frac{x-3}{4} + \frac{x+2}{5}$ c $\frac{2x}{3} - \frac{5x}{9}$

d $\frac{1}{x+2} + \frac{1}{x+3}$ e $\frac{4}{x+2} - \frac{3}{x+1}$ f $\frac{1}{2x-1} - \frac{1}{2x+3}$

Example 8 Write $\frac{1}{x} - \frac{3}{x^2+3x}$ as a single fraction in its simplest form.

$\frac{1}{x} - \frac{3}{x^2+3x} = \frac{1}{x} - \frac{3}{x(x+3)}$ ← Factorise the denominator $x^2 + 3x$.

Common denominator $= x(x+3)$ ← Find the lowest common denominator.

$\frac{1}{x} - \frac{3}{x^2+3x} = \frac{x+3}{x(x+3)} - \frac{3}{x(x+3)}$ ← Write each fraction with the same denominator.

$= \frac{x+3-3}{x(x+3)}$ ← Combine the fractions.

$= \frac{x}{x(x+3)}$ ← Simplify the numerator.

$= \frac{1}{x+3}$ ← Divide the numerator and denominator by x to simplify your answer.

ResultsPlus Examiner's Tip

$x \div x = 1$

$x(x+3) \div x = x + 3$

Example 9 Simplify $\frac{1}{5x+10} + \frac{1}{x^2+5x+6}$

$5x + 10 = 5(x+2)$
$x^2 + 5x + 6 = (x+3)(x+2)$ ← Factorise each denominator.

$\frac{1}{5x+10} + \frac{1}{x^2+5x+6} = \frac{1}{5(x+2)} + \frac{1}{(x+3)(x+2)}$ ← Replace the denominators with the factorised expressions.

Common denominator $= 5(x+3)(x+2)$ ← Find the lowest common denominator of $5(x+2)$ and $(x+3)(x+2)$.

$\frac{1}{5x+10} + \frac{1}{x^2+5x+6} = \frac{(x+3)}{5(x+3)(x+2)} + \frac{5}{5(x+3)(x+2)}$ ← Write equivalent fractions with the common denominator.

$= \frac{x+3+5}{5(x+3)(x+2)}$ ← Combine the fractions.

$= \frac{x+8}{5(x+3)(x+2)}$

Exercise 32C

D

1 a Factorise i $2x + 2$ ii $6x + 6$.
b Write down the lowest common multiple of $2x + 2$ and $6x + 6$.
c Write $\frac{1}{2x + 2} + \frac{1}{6x + 6}$ as a single fraction in its simplest form.

B

2 Write down the lowest common multiple of each of the following pairs of expressions.
a $3x$ and $5x$
b $x + 2$ and $x + 3$
c x and $x(x - 1)$
d $x + 2$ and $(x + 1)(x + 2)$
e $2x - 6$ and $x - 3$
f $x + 1$ and $x^2 + x$

A*

3 a Factorise $x^2 + 3x + 2$.
b Write $\frac{1}{x + 2} - \frac{1}{x^2 + 3x + 2}$ as a single fraction in its simplest form.

4 a Factorise $x^2 - 4$.
b Write $\frac{3}{x - 2} - \frac{2}{x^2 - 4}$ as a single fraction in its simplest form.

5 a Factorise $2x^2 - 3x + 1$.
b Write $\frac{1}{2x^2 - 3x + 1} + \frac{2}{2x - 1}$ as a single fraction in its simplest form.

6 Simplify $\frac{1}{2x + 6} - \frac{1}{x^2 + 4x + 3}$

7 Write $\frac{1}{4} + \frac{1}{2x} + \frac{1}{8(x + 1)}$ as a single fraction.

8 Express $\frac{3}{3 - x} - \frac{9}{9 - x^2}$ as a single fraction.

9 a Factorise i $x^2 + 9x + 20$ ii $x^2 + 11x + 30$.
b Write $\frac{4}{x^2 + 9x + 20} - \frac{1}{x^2 + 11x + 30}$ as a single fraction in its simplest form.

10 Show that $\frac{1}{4x^2 - 8x + 3} - \frac{1}{4x^2 - 1} = \frac{A}{(2x - 1)(2x + 1)(2x - 3)}$ and find the value of A.

32.3 Multiplying and dividing algebraic fractions

Objective

- You can multiply and divide algebraic fractions.

Why do this?

Doctors and nurses need to multiply and divide algebraic fractions when calculating the drugs dosage to give their patients.

Get Ready

1. Simplify.

a $\frac{ab}{b}$ b $\frac{x + 1}{2x + 2}$ c $\frac{(x + 1)(x + 3)}{(x + 1)^2}$

Key Points

- To multiply (or divide) algebraic fractions we use a similar method to that used for multiplying and dividing numerical fractions.
- To multiply fractions, multiply the numerators and multiply the denominators.
  $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
- To divide fractions, multiply the first fraction by the reciprocal of the second.
  $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$
- Simplify your answers if you can.
- You may need to factorise the numerator and/or the denominator before you multiply or divide algebraic fractions.

Example 10 Simplify $\frac{2x}{3} \times \frac{x}{4}$

$\frac{2x}{3} \times \frac{x}{4} = \frac{2x \times x}{3 \times 4} = \frac{{}^{1}\cancel{2}x^2}{\cancel{12}_{6}}$ ← Multiply $2x$ by x. Work out 3×4.

$= \frac{x^2}{6}$ ← Divide both the numerator and denominator by 2.

Example 11 Simplify $\frac{2x}{5y} \div \frac{x^2}{y}$

$\frac{2x}{5y} \div \frac{x^2}{y} = \frac{2x}{5y} \times \frac{y}{x^2}$ ← Multiply the first fraction by the reciprocal of the second.

$= \frac{2 \times \cancel{x}^{1} \times \cancel{y}^{1}}{5 \times \cancel{y}_{1} \times \cancel{x}_{1} \times x}$ ← Divide both the numerator and denominator by x and by y.

$= \frac{2}{5x}$

Example 12 Simplify $\frac{x+1}{x+2} \times \frac{(x+2)^2}{3(x+1)}$

$\frac{x+1}{x+2} \times \frac{(x+2)^2}{3(x+1)} = \frac{\cancel{(x+1)}^{1}\cancel{(x+2)}^{1}(x+2)}{3\cancel{(x+2)}_{1}\cancel{(x+1)}_{1}}$ ← Write down the *product* of the two numerators and the product of the two denominators.

$= \frac{x+2}{3}$ ← Simplify the fraction.

Example 13 Simplify $\frac{2x-1}{4} \div \frac{4x-2}{5}$

$\frac{2x-1}{4} \div \frac{4x-2}{5} = \frac{2x-1}{4} \div \frac{2(2x-1)}{5}$ ← Factorise the numerator of the second fraction.

$= \frac{2x-1}{4} \times \frac{5}{2(2x-1)}$

$= \frac{5\cancel{(2x-1)}^{1}}{8\cancel{(2x-1)}_{1}}$ ← Divide the numerator and denominator by $2x - 1$.

$= \frac{5}{8}$

Example 14 Simplify $\frac{2x+1}{x^2-1} \times \frac{x+1}{2x^2-x-1}$

$$\frac{2x+1}{x^2-1} \times \frac{x+1}{2x^2-x-1} = \frac{2x+1}{(x-1)(x+1)} \times \frac{x+1}{(2x+1)(x-1)}$$

Factorise $x^2 - 1$ and $2x^2 - x - 1$.

$$= \frac{(2x+1)(x+1)}{(x-1)(x+1)(2x+1)(x-1)}$$

Write down the product of the two fractions and simplify.

$$= \frac{1}{(x-1)(x-1)}$$

$$= \frac{1}{(x-1)^2}$$

Write $(x-1)(x-1)$ as $(x-1)^2$.

ResultsPlus Examiner's Tip

Note that it is often preferable to leave the fraction in factorised form.

Exercise 32D

C

1 Write as a single fraction.

a $\frac{x}{3} \times \frac{x}{5}$ b $\frac{4}{y} \times \frac{3}{y}$ c $\frac{5x}{2} \times \frac{3y}{4}$ d $\frac{x}{3} \times \frac{x-3}{4}$

B

2 Write as a single fraction in its simplest form.

a $\frac{3x}{5} \times \frac{10y}{12}$ b $\frac{4x}{9y} \times \frac{y}{2}$ c $\frac{2x^2}{y^2} \times \frac{3y}{x^2}$ d $\frac{x+1}{x} \times \frac{2x}{x-1}$

3 Write as a single fraction.

a $\frac{x}{9} \div \frac{x}{5}$ b $\frac{5x}{6} \div \frac{3}{y}$ c $x^2y \div \frac{1}{y}$ d $\frac{2x}{x+1} \div \frac{x+1}{x+2}$

4 Write as a single fraction in its simplest form.

a $\frac{3x}{2} \div \frac{2x}{9}$ b $\frac{5y^2}{8x} \div \frac{y^2}{x^2}$ c $\frac{7x}{12y} \div \frac{y}{6}$ d $\frac{x}{2} \div \frac{x-5}{4}$

A

5 Write as a single fraction in its simplest form.

a $\frac{x+1}{3} \times \frac{3x+3}{2}$ b $\frac{x+2}{x-1} \times (x-1)^2$ c $\frac{x}{x+1} \times \frac{x+1}{x+2}$

d $\frac{x+4}{9} \div \frac{2x+8}{3}$ e $\frac{6}{3x-1} \div \frac{2}{(3x-1)^2}$ f $\frac{3x-12}{4} \div \frac{x-4}{x+4}$

A*

6 a Factorise $x^2 - 4$.

b Write $\frac{1}{x+2} \times \frac{x^2-4}{x^2+4}$ as a single fraction in its simplest form.

7 a Factorise i $x^2 + 5x + 4$ ii $x^2 + 6x + 8$.

b Write $\frac{x+3}{x^2+5x+4} \div \frac{x+1}{x^2+6x+8}$ as a single fraction in its simplest form.

8 Write $\frac{x^2-x}{x^2+x} \times \frac{x^2+2x+1}{x-1}$ as a single fraction in its simplest form.

32.4 Algebraic proof

Objective

- You can prove a given result using algebra.

Why do this?

Algebraic proofs are used to prove many key ideas (theorems) in life, from thermodynamics to quantum mechanics.

Get Ready

1. n is an integer.
State whether each of the following must represent an even number, an odd number or either.
a $2n$ **b** $n + 1$ **c** $2n - 1$ **d** n^2 **e** $3n$

Key Points

- To demonstrate that a result is true, you can give details of a particular case. For example, to demonstrate that the sum of two odd numbers is even, you could choose any two odd numbers and show that their sum is even, such as $3 + 5 = 8$.
- To **prove** that a result is true, you must show that it will be true in all cases. For example, to prove that the sum of two odd numbers is even, you must choose two 'general' odd numbers and show that their sum will always be even. You could write
$(2m - 1) + (2n - 1) = 2m + 2n - 2 = 2(m + n - 1)$ which is an even number as it is a multiple of 2.
- In algebraic **proof** you will find the following points helpful.
Where n is an integer:
  - Consecutive integers can be written in the form $n, n + 1, n + 2, n + 3, \ldots$ In some cases it is more useful to write them in a slightly different form, for example $n - 2, n - 1, n, n + 1, n + 2, \ldots$
  - Any even number can be written in the form $2n$.
  - Consecutive even numbers can be written in the form $2n, 2n + 2, 2n + 4, \ldots$
  - Any odd number may be written in the form $2n - 1$ (alternatively any odd number may be written in various other forms, for example $2n + 1$).
  - Consecutive odd numbers can be written in the form $2n - 1, 2n + 1, 2n + 3, \ldots$

Example 15

a Show that $(2n - 1)^2 + (2n + 1)^2 = 8n^2 + 2$.
b Hence prove that the sum of the squares of any two consecutive odd numbers is even.

a $(2n - 1)^2 + (2n + 1)^2 = (2n - 1)(2n - 1) + (2n + 1)(2n + 1)$ ← Write out the expression.

$= 4n^2 - 4n + 1 + 4n^2 + 4n + 1$ ← Multiply out the brackets.

$= 8n^2 + 2$ ← Simplify the expression.

b Any odd number may be written in the form $2n - 1$, where n is an integer. The next odd number can therefore be written as $2n + 1$. ← Add 2 to $2n - 1$.

So the sum of the squares of any two consecutive odd numbers $= (2n - 1)^2 + (2n + 1)^2$. ← Write the sum of the squares of any two consecutive odd numbers algebraically.

$= 8n^2 + 2$ ← Use the identity given in part **a**.

$= 2(4n^2 + 1)$ ← Show that 2 is a factor of the expression.

As $8n^2 + 2$ is a multiple of 2 it must represent an even number. ← Explain why the expression must represent an even number.

So, the sum of the squares of any two consecutive odd numbers is always even. ← Complete your proof by stating the result.

Exercise 32E

A* A03

1. Prove that the sum of any odd number and any even number is odd.
2. * Prove that half the sum of four consecutive numbers is odd.
3. * Prove that the sum of any three consecutive numbers is a multiple of 3.
4. **a** Prove that the product of any odd number and any even number is even.
   b Prove that the product of any two odd numbers is odd.
   c Prove that the product of any two even numbers is even.
5. * Prove that for any two numbers the product of their difference and their sum is equal to the difference of their squares.
6. * Prove that, if the difference of two numbers is 4, then the difference of their squares is a multiple of 8.

Chapter review

- Algebraic fractions, like numerical fractions, can often be simplified.
- To simplify an algebraic fraction, factorise the numerator and the denominator, then divide the numerator and denominator by any common factors.
- To add (or subtract) algebraic fractions we use a similar method to that used for adding and subtracting numerical fractions.
- If the denominator of the fractions are the same, add (or subtract) the numerators but do not change the denominator.
- To add (or subtract) algebraic fractions with different denominators, find a common denominator and write each fraction as an equivalent fraction with this denominator.

- To find the lowest common denominator of algebraic fractions, you may need to factorise the denominators first.
- To simplify your answers you may have to factorise the numerator.
- To multiply (or divide) algebraic fractions we use a similar method to that used for multiplying and dividing numerical fractions.
- To multiply fractions, multiply the numerators and multiply the denominators.
  $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$
- To divide fractions, multiply the first fraction by the reciprocal of the second.
  $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$
- You may need to factorise the numerator and/or the denominator before you multiply or divide algebraic fractions.
- To demonstrate that a result is true, you can give details of a particular case.
- To **prove** that a result is true, you must show that it will be true in all cases.
- In algebraic **proof** you will find the following points helpful.
  Where n is an integer:
  - Consecutive integers can be written in the form $n, n + 1, n + 2, n + 3, \ldots$ In some cases it is more useful to write them in a slightly different form, for example $n - 2, n - 1, n, n + 1, n + 2, \ldots$
  - Any even number can be written in the form $2n$.
  - Consecutive even numbers can be written in the form $2n, 2n + 2, 2n + 4, \ldots$
  - Any odd number may be written in the form $2n - 1$ (alternatively any odd number may be written in various other forms, for example $2n + 1$).
  - Consecutive odd numbers can be written in the form $2n - 1, 2n + 1, 2n + 3, \ldots$

Review exercise

A

1 Here are the first 4 lines of a number pattern.

1 + 2 + 3 + 4	=	(4 × 3) − (2 × 1)
2 + 3 + 4 + 5	=	(5 × 4) − (3 × 2)
3 + 4 + 5 + 6	=	(6 × 5) − (4 × 3)
4 + 5 + 6 + 7	=	(7 × 6) − (5 × 4)

n is the first number in the nth line of the number pattern.
Show that the above number pattern is true for the four consecutive integers

$n, (n + 1), (n + 2)$ and $(n + 3)$

Nov 2007

A*

2 Simplify fully.

a $\frac{4x^3}{8x}$ b $\frac{2(x + 1)^4}{6(x + 1)^2}$

c $\frac{x^2 + 5x}{x + 5}$ d $\frac{x^2 + 7x + 6}{x^2 + 8x + 12}$

e $\frac{x^2 - 2x}{x^2 + x - 6}$ f $\frac{x^2 - 25}{x^2 + 10x + 25}$

g $\frac{x^2 - 2x + 1}{x^3 - x}$ h $\frac{2x^2 + 7x - 4}{6x^2 + x - 2}$

ResultsPlus
Exam Question Report

78% of students answered this sort of question poorly because they simplified at the wrong stage in the calculation.

3 Write as a single fraction in its simplest form.

a $\frac{3x}{10} + \frac{x}{5}$ **b** $\frac{3}{2x} + \frac{2}{3x}$ **c** $\frac{1}{5x - 3} + \frac{1}{5x + 3}$

d $\frac{3}{(x + 1)(x + 2)} + \frac{2}{x + 1}$ **e** $\frac{1}{(x + 1)^2} + \frac{1}{x + 1}$ **f** $\frac{1}{x^2 + 4x + 3} + \frac{1}{x^2 + 8x + 15}$

4 Simplify.

a $\frac{x}{2} \times \frac{4}{x}$ **b** $\left(\frac{x}{3}\right)^2 \times \frac{9}{x}$ **c** $\frac{x}{10} \div \frac{x}{4}$ **d** $\frac{2s}{3} \div \frac{1}{s}$

e $\frac{x - 1}{(x + 1)^2} \times \frac{x + 1}{x + 3}$ **f** $\frac{x^2 - x}{x^2 + x} \times \frac{x^2 - 1}{(x - 1)^2}$ **g** $\frac{x + 2}{5} \div \frac{3x + 6}{10}$ **h** $\frac{2x - 1}{x^2} \div \frac{2x - 1}{2x^2 + x}$

AO3 **5** **a** Show that $(n + 1)(n + 2) + n(n + 1) = 2(n + 1)^2$.

b Hence prove that for any three consecutive integers, the sum of the product of the last two and the product of the first two is always even.

AO3 * **6** Prove that the difference of the squares of two consecutive odd numbers is a multiple of 8.

AO3 **7** Show that $25 - \frac{(x - 8)^2}{4} = \frac{(2 + x)(18 - x)}{4}$ *June 2005*

AO3 **8** The nth even number is $2n$.

a Explain why the next even number after $2n$ is $2n + 2$.

b Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6.

ResultsPlus
Exam Question Report

93% of students answered this question poorly because they did not follow the direction indicated in the early parts of the question.

June 2008, adapted

AO3 * **9** Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 4, for all positive integer values of n. *June 2009*

AO2 AO3 **10** Jim runs 16 km from home at a speed of x kph. He then runs the same distance back home at a speed 1 kph slower.
Work out an expression, in terms of x, for the total time Jim took to run from home and back.
Give your answer as a single fraction in its simplest form.